

SIMULATIONS OF CLUSTERS OF GALAXIES

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Numerical simulations of clusters of galaxies provide a unique way to follow the dynamics of these systems. The models reveal many characteristics of the merging process of subclusters: shock structure and strength, temperature distribution and gas distribution. From the models detailed observational signatures of the dynamical state can be derived. The simulations also show that mergers have effects on the magnetic field, on the X-ray luminosity, on the metal enrichment and other physical processes. Furthermore, observational methods like the mass determination can be tested.

1 Introduction

Clusters of galaxies evolve on time scales of the order or little less than the age of the universe. Hence with observations one cannot follow their evolution. Observations provide only snapshots of the different evolutionary stages. Therefore only numerical simulations provide the possibility to follow the dynamics of clusters.

Numerical models cannot only be used to learn how the clusters form and evolve, but also observable quantities can be derived, which can be compared directly with observations. This comparison is used for many different purposes. Observations can be interpreted more easily, e.g. the models can distinguish what observable feature corresponds to which dynamical state. Moreover, observational methods can be tested with the simulated data. The results can be compared to the input parameters of the calculations and in an iterative way the methods can be refined. In this way the physical processes in clusters and cluster evolution can be understood. This knowledge provides the opportunity to constrain cosmological models.

Merging of subclusters are particularly interesting phenomena to study with simulations. Irregular cluster morphologies in many X-ray images of cluster as well as indications from optical observations imply that many clusters are not relaxed. Hence mergers are very common in clusters of galaxies. Such mergers of subclusters are very energetic events, which affect clusters strongly, e.g. the cause shocks. These shocks are the major heating source for the intra-cluster gas and also particles can be re-accelerated there to relativistic energies.

This article is organised as follows. In Sect. 2 the different simulation methods are explained. Sect. 3 shows what can be learned from models in particular about the merger process of clusters of galaxies. The effects of magnetic fields and other physical processes like radiative cooling and star formation are discussed in Sections 4 and 5, respectively. In Sect. 6 results of simulations of different metal enrichment processes are presented. Sect. 7 shows how useful numerical models are to test the mass determination method. A short summary is given in Sect. 8.

2 Simulation Methods

For realistic simulations three-dimensional calculations are required. Also the different cluster components must be taken into account: dark matter, galaxies and intra-cluster gas. Dark matter and galaxies can be regarded as collisionless particles and can therefore be modelled by N-body simulations. In these kinds of simulations only the gravitational interaction between the particles are taken into account. Each particle is moved in the force field of all the other particles. For current particle numbers of 128^3 or 256^3 this procedure would take a lot of computing time to calculate the force by simply summing over all the other particles' contributions. To accelerate the calculations different techniques have been developed, e.g. the particles are sorted onto a grid or into a tree structure. In this way several particles are combined and treated simultaneously without losing much accuracy but gaining a lot of computing time. Many simulations have been performed which take into account only dark matter and apply therefore only N-body calculations. Such kind of simulations are very useful for many purposes because the dark matter makes up most of the gravitative mass. In this article, however, I will concentrate only on models which include both, the dynamics of the dark matter and of the gas.

For the simulation of the gas pressure must be taken into account, i.e. the full hydrodynamic equations must be solved. Two different methods have been used generally for these hydrodynamic calculations: (1) Smoothed Particle Hydrodynamics (SPH; Lucy 1977; Monaghan 1985): This is a Lagrangian approach, i.e. the calculation follows the fluid. The gas is treated as particles in this approach. Examples of this type of simulations are: Evrard (1990), Dolag et al. (1999), Takizawa (1999), and Takizawa & Naito (2000). (2) Grid-based codes: this is the Eulerian approach, i.e. the simulation volume is divided into cells and the fluid is moving in this grid which is fixed in space. Examples of simulations using grid codes can be found in Schindler & Müller (1993), Bryan et al. (1994), Roettiger et al. (1997), Ricker (1998) and Quilis et al. (1998).

Fortunately, the choice of simulation technique is not essential. Calculations with both methods yield very similar results. This was tested in a large project, the Santa Barbara Cluster Comparison Project (Frenk et al. 1999), in which the formation of a galaxy cluster was simulated using 12 different techniques. Both methods, SPH and grid codes, were applied. Each simulation started with exactly the same initial conditions. The comparison showed very good agreement in the properties of the dark matter. Also relative good agreement was found in the gas temperature, the gas mass fraction and the gas profiles of the final cluster. The largest discrepancies were found in the X-ray luminosity which differed by up to a factor of 2.

3 Cluster Models

In general cluster models can reproduce cluster morphologies and parameters known from observations quite well. The temperatures of the X-ray emitting intra-cluster gas are typically very well simulated. Also the spatial distributions are very realistic. For the dark matter component a NFW profile (Navarro et al. 1995) is usually found, while the gas profile is well fit by a so-called β -profile (Cavaliere & Fusco-Femiano 1976).

The models can distinguish between cosmological parameters. Simulations on large scales show distinctly different distribution of matter for a mean density $\Omega_M = 1$ and $\Omega_M = 0.3$, respectively (Ostriker & Cen 1996; Thomas et al. 1998; Jenkins et al. 1998). While in the latter model the distribution changes only slightly between a redshift $z=1$ and now, in the $\Omega_M = 1$ model significant differences are visible in the same time interval: many smaller structures merge to larger structures, so that the distribution looks much less smooth at $z=0$ than at $z=1$. A distinction of $\Lambda = 0$ models and $\Lambda \neq 0$ models is difficult though.

3.1 Mergers of Clusters

Simulations are ideal to follow the different stages of a merging event (see Fig. 1). In the pre-merger phase the two subclusters are approaching each other and can still be seen as well separated units. The collision, i.e. the moment when the cores pass through each other, is characterized by enhanced X-ray luminosity and enhanced temperature. In the subsequent post-merger stage shock waves emerge – mainly in direction of the original collision axis. The shocks propagate outwards and are visible for about 1 - 2 Gyrs.

3.2 Shocks

The most prominent features emerging from mergers are shock waves in the intra-cluster gas resulting from colliding subclusters with relative velocities of up to ≈ 3000 km/s. When a dense subcluster falls into a cluster a shock emerges already before the core passage: a bow shock is visible in front of the infalling subcluster (Roettiger et al. 1997). The strongest shocks emerge after the collision of subclusters, when these shocks propagate (Schindler & Müller 1993; Roettiger et al. 1999a; see Fig. 1e). These shocks are relatively mild shocks, though, with a maximum Mach number of about 3. The shocks are visible as steep gradients in the gas density and in the gas temperature. In general, the shock structure is found to be more filamentary at early epochs of cluster formation and quasi-spherical at low redshifts (Quilis et al. 1998).

Observationally, the shocks are best visible in X-ray temperature maps, because they show up as steps in these maps (Fig. 1e; Schindler & Müller 1993). For such maps spatially resolved X-ray spectroscopy is necessary which can be performed now with high accuracy with the new X-ray observatories XMM and Chandra.

The shocks are not only the major heat source of the intra-cluster gas, but they are also of particular interest for particle acceleration models. It is likely that particles are re-accelerated to relativistic energies in these shocks. These relativistic particles are responsible for the non-thermal emission and their interaction with the cluster magnetic field shows up as synchrotron emission in radio halos (e.g. Giovannini & Feretti 2001; Feretti 2001).

The shocks heat primarily the ions as has been shown in simulations which treat ions and electrons separately (Chièze et al. 1998; Takizawa 1999). Only later the energy is transferred to the electrons.

3.3 Observational Signatures

Apart from shocks, there are several other signatures which can be used to determine the dynamical state of a galaxy cluster. For example, the X-ray luminosity increases during the collision of two subclusters (Schindler & Müller 1993). The reason is that the gas is compressed, i.e. the gas density is increased, and as the X-ray emission is proportional to the square of the density we see enhanced X-ray emission during the core passage of two subclusters. Shortly before the collision the gas between the subclusters is heated due to compression and shows up as a high temperature region (see Fig. 1c).

During the core passage and at each rebound an increase in the magnetic field is visible (Dolag et al. 1999; Roettiger et al. 1999b; see also Section 4). Mergers also cause turbulence and motion in the intra-cluster gas. Off-centre collisions produce additionally angular momentum (Ricker 1998; Roettiger et al. 1998).

Observationally, mergers cannot only be identified by multiple X-ray maxima, but also by isophote twisting with centroid shift and elongations: the collisionless component is always elongated along the collision axis, before and after the collision. The gas is first elongated along the collision axis. During the core passage it is pushed out perpendicular to the collision axis, so that later an elongation perpendicular to the collision axis can be seen (Schindler & Müller

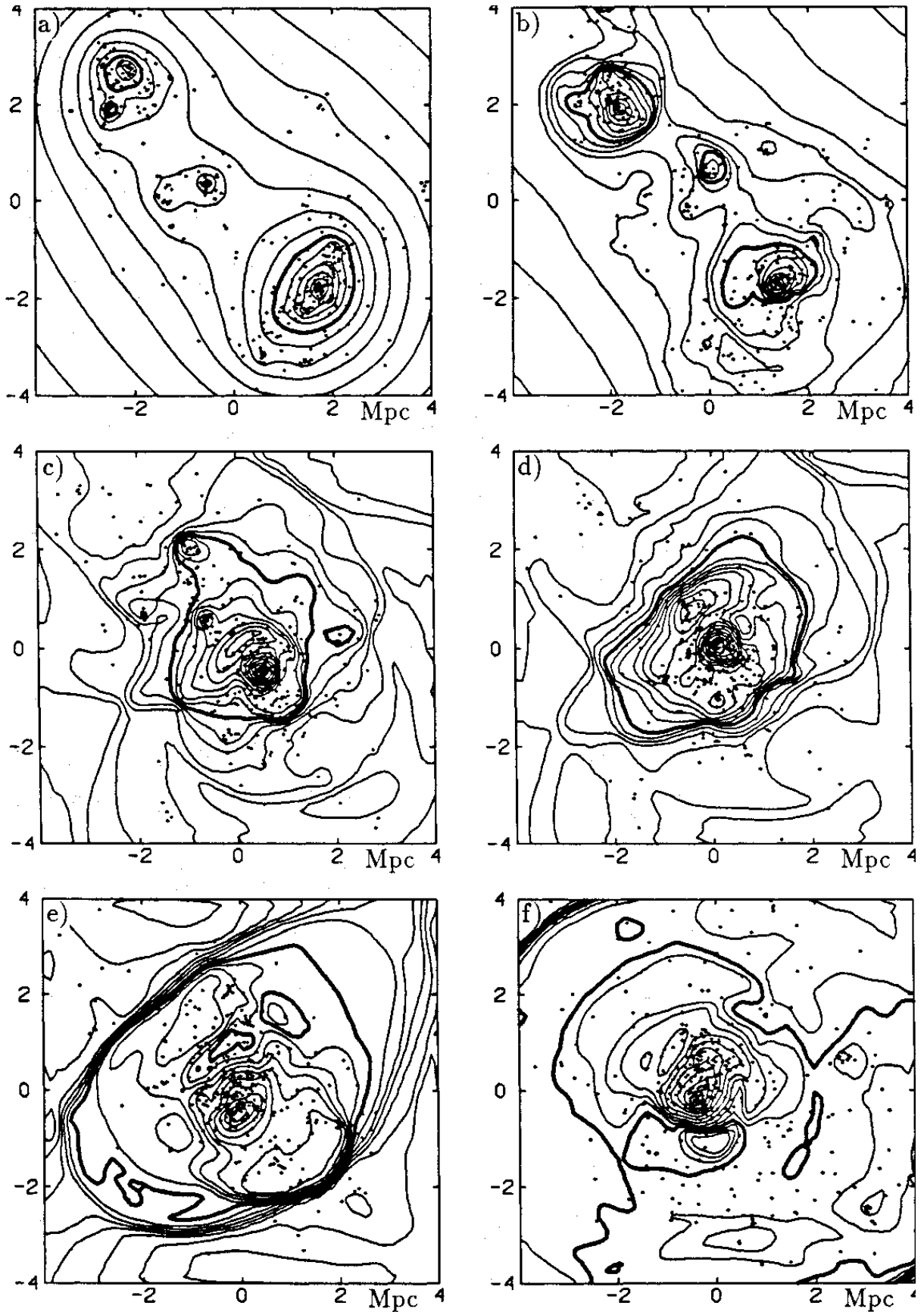


Figure 1: Evolution of the X-ray temperature. The temperature contours are logarithmically spaced with $\Delta \log T = 0.05$ the bold contour line corresponding to a temperature of 10^8 K. The six snapshots are taken at b) $t = 0.95$ Gyr, c) $t = 2.7$ Gyr, d) $t = 3.2$ Gyr, e) $t = 4.4$ Gyr, f) $t = 6.1$ Gyr after the configuration shown in a).

1993; see Fig. 1e). Also offsets between the collisionless component and the gas have been found (Roettiger et al. 1997). If more than one merger occurs in a relatively short time interval the temperature structure can become very complex.

4 Cluster Simulations with Magnetic Fields

Faraday rotation measurements indicate that clusters are permeated by magnetic fields of the order of $1 \mu\text{G}$ (e.g. Kim et al. 1991). Also radio halos require the existence of magnetic fields in clusters on scales of a few Mpc (e.g. Giovannini et al. 1991, 1993). Therefore magneto-hydrodynamic calculations have been performed (Dolag et al. 1999; Roettiger et al. 1999b; see also Dolag, this volume) to investigate the origin, the distribution and the evolution of the magnetic fields (see Fig. 2). Results of these simulations are: the initial field distribution is irrelevant for the final structure of the magnetic field. The structure is dominated only by the cluster collapse. Faraday rotation measurements can be reproduced by the simulations for magnetic fields of the order of $1 \mu\text{G}$ in very good agreement with the value inferred from observations.

Most important for the amplification of the magnetic field are shear flows, while the compression of the gas is of minor importance. Mergers of subclusters change the local magnetic field strength as well as the structure of the cluster-wide field. At early stages of the merger, filamentary structures prevail, which break down later and leave a stochastically ordered magnetic field.

5 Cooling and Star Formation

It might be necessary to take into account additional physical processes in order to achieve realistic models. Two important processes are star formation and radiative cooling.

In simulations without cooling and star formation it was found, that the gas is less concentrated than dark matter. Also the X-ray luminosity - temperature relation inferred from simulations is in disagreement with observations (Eke et al. 1998, Bryan & Norman 1998, Yoshikawa et al. 2000). The question arises, whether this is a numerical artifact due to the negligence of physical processes or due to the difficulty in determining the X-ray luminosity correctly from the numerical models. Another possibility is that it is connected with observational findings of different profiles of baryonic and dark matter (Schindler 1999) and the deviation of the X-ray luminosity - temperature relation from a pure power law (Ponman et al. 1999) which could both be explained by non-gravitational heating processes. In order to test this hypothesis several groups have performed simulations with cooling and star formation and came to quite different conclusions.

Lewis et al. (2000) found that models with cooling and star formation have a 20% higher X-ray luminosity and a 30% higher temperature in the cluster centre. Also Sugimotohara & Ostriker (1998) find that radiative cooling increases luminosity. In contrast to these results Pearce et al. (2000) and Muanwong et al. (2001) find that radiative cooling decreases the total X-ray luminosity.

In order to test whether the gas is less concentrated than the dark matter because of pre-heating, i.e. early non-gravitational heating, Bialek et al. (2000) performed simulations with an initially elevated adiabat. They find that they can reproduce the observations when adding an initial entropy of $55 - 150 \text{ keV cm}^2$.

Mathiesen & Evrard (2001) took a different approach and tested with their models how good the observational temperature determination is. They simulated CHANDRA spectra and found that the temperature can be underestimated by up to 20% by the standard temperature determination method. The reason is cold material falling onto the cluster from all sides, i.e.

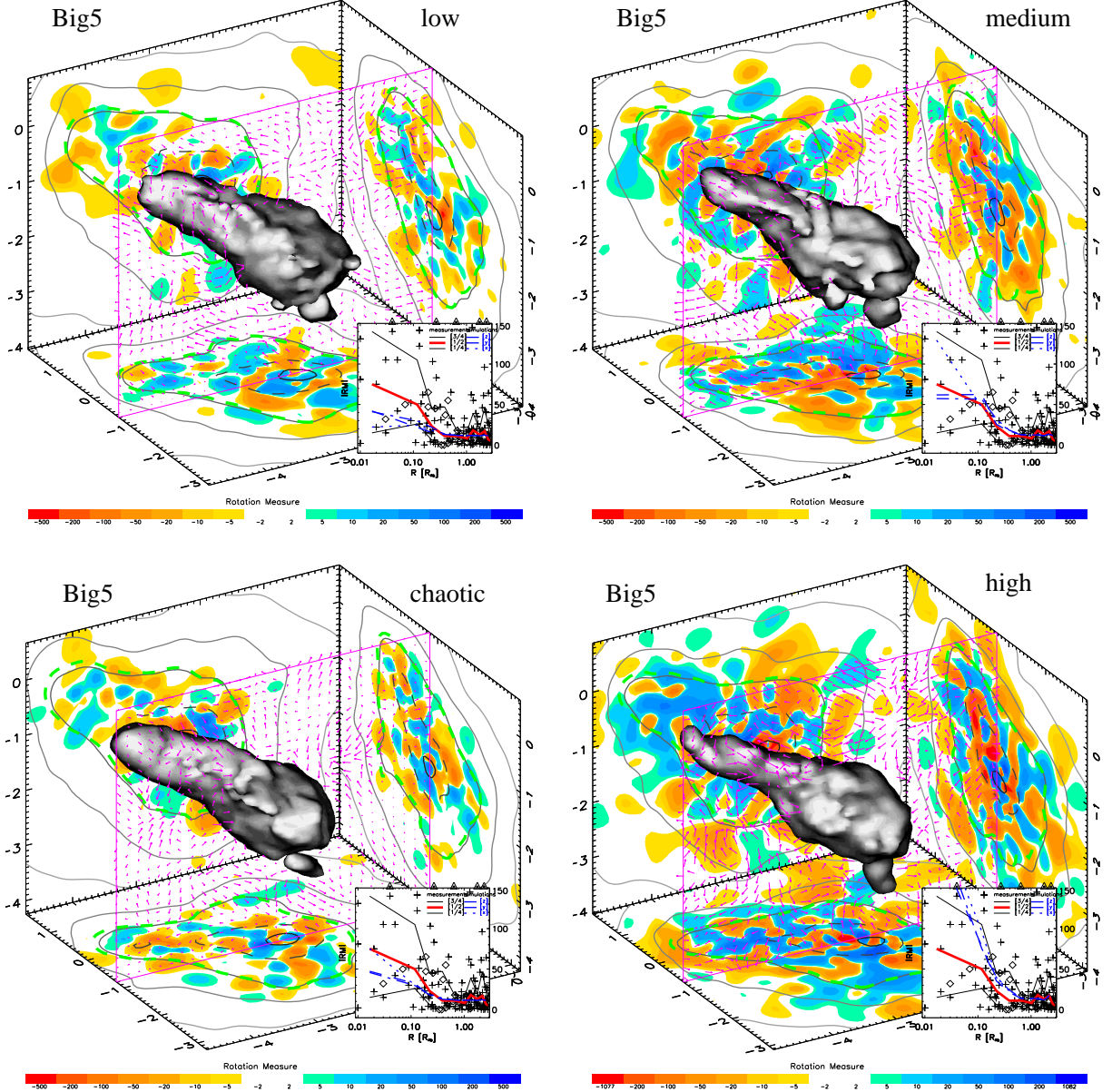


Figure 2: Isodensity surface of four cluster models with magnetic fields in the core region of $0.4\mu G$ (“low” and “chaotic”), $1.1\mu G$ (“medium”) and $2.5\mu G$ (“high”), respectively, at redshift $z=0$. The three projections show the rotation measure (colour) and the gas density (contours). The purple arrows indicate the orientation and the strength of the magnetic field in a plane containing the cluster centre. The insets show the rotation measure versus distance from the cluster centre: simulations (lines) and observations in several clusters (symbols) (from Dolag (2000)).

also along the line of sight, resulting in a cold contribution to the cluster spectrum and hence to a lower temperature determination.

Bryan & Norman (1998) showed that in simulations the mass - temperature relation is much more robust than the X-ray luminosity - temperature relation and hence suggested to use the former relation for drawing conclusions about the cluster formation process.

The star formation rate can be affected by mergers of subclusters. It is still controversial whether mergers increase or decrease the star formation activity. The interstellar medium in a galaxy can be compressed during a merger, which would lead to an increased star formation rate. This effect was predicted by simulations by Evrard (1991). Also in a number of observations a connection between mergers and enhanced star formation rate has been found. In contrast to these results Fujita et al. (1999) found in simulations that the interstellar medium in the galaxies is stripped off due to increased ram pressure during the merger, so that the galaxies contain less gas and therefore show decreased star formation activity. At the moment of the subcluster collision, they found an increase of post-starburst galaxies which indicates that a rapid drop in the star formation rate must have occurred.

6 Metal Enrichment

The intra-cluster gas contains metals. This implies that the gas cannot be purely of primordial origin, but it must at least partially have been processed in the cluster galaxies and have been expelled from the galaxy potential into the intra-cluster medium. As mentioned above the star formation activity, i.e. the metal production rate, and its connection to the dynamical state is still controversial. Also the gas ejection processes and their time scales are still under discussion. Several gas ejection processes have been suggested: e.g. ram-pressure stripping (Gunn & Gott 1972), galactic winds (De Young 1978), galaxy-galaxy interaction and jets from active galaxies.

In order to decide which process is dominating at what time it is necessary to compare observations and simulations in particular with respect to the metallicity distribution within clusters and to the metallicity evolution with redshift.

The effects of supernova driven winds were studied by many groups. David et al. (1991) calculated the first models on cluster scales. They found that the results depend sensitively on the input parameters: the stellar initial mass function, the adopted supernova rate and the primordial fraction of intra-cluster gas. In 3D models calculating the gas dynamics and galactic winds Metzler & Evrard (1994, 1997) found that winds can account for the observed metal abundances. They find very strong metallicity gradients (almost a factor of ten between cluster centre and virial radius) which are hardly affected by cluster mergers. From simulations on galaxy scale Murakami & Babul (1999) concluded that galactic winds are not very efficient for the metal enrichment.

Another process which is probably important for the metal enrichment is ram-pressure stripping: as a galaxy approaches the cluster centre it experiences an increasing pressure and at some point the galaxy potential is not strong enough to retain the galaxy gas. The gas is stripped off and the metals are released into the intra-cluster medium.

Simulations of ram-pressure are relatively difficult because not only the conditions of the gas inside the galaxy and the potential of the galaxy must be taken into account, but also the conditions of the surrounding medium. Recently, high resolution simulations were carried out to study the stripping process in different types of galaxies. Abadi et al. (1999) and Quilis et al. (2000) performed simulations of spiral galaxies. They found that their gas can be stripped off when it is not homogeneous. Details of the spatial distribution and the time dependence of the process are shown. For dwarf galaxies Mori & Burkert (2000) found in their simulations that the gas is easily stripped off when these galaxies move through the intra-cluster medium. In simulations of elliptical galaxies (Tonozzo & Schindler 2001) showed details of the stripping

process of these galaxies. The amount of gas stripped off depends sensitively on the orbit of the galaxy. The gas cannot only be stripped off as the galaxy approaches the cluster centre, but the galaxy can again accumulate some gas when it is in the apocentre of its orbit. X-ray morphologies of the simulated galaxies are derived as well.

All these simulations showed that ram-pressure stripping can be an important metal enrichment process. Merging activity increases the effect even more because the ram pressure is proportional to $\rho_{ICM} \times v_{rel}^2$ with v_{rel} being the relative velocity of intra-cluster gas and galaxies. During mergers not only the gas density is increased but also the relative velocities are much higher than in a relaxed cluster. Therefore a large influence of the merging processes on the stripping rate are expected.

7 Mass Determination of Clusters of Galaxies

Numerical models are ideal tools to test observational methods. An example for such a method is the mass determination from X-ray observations. In this method the X-ray emitting gas is used as a tracer for the potential. Mass determinations are very important because they measure in an indirect way the amount and the distribution of dark matter in clusters.

With the assumption of hydrostatic equilibrium and spherical symmetry the total cluster mass can be expressed in terms of only two observable quantities, which can both be inferred from X-ray observations: the gas temperature and the gas density.

The total mass of a model cluster is known exactly for each time step and within any radius. On the other hand the X-ray emission of the model cluster can be simulated and the normal X-ray mass determination method can be applied to these simulated X-ray data. A comparison of the true mass and the X-ray mass yields the accuracy of the X-ray method.

As shown by several groups the X-ray mass determination method proved to be quite reliable in relaxed clusters (Evrard et al. 1996; Roettiger et al. 1996; Schindler 1996). Typical errors are 15% without any preference for over- or underestimation.

Only during mergers quite strong deviations can occur. The reason is that the two assumptions for the method – hydrostatic equilibrium and spherical symmetry are not well obeyed during mergers. For example, at the location of shocks the gas is not in hydrostatic equilibrium. Shocks cause gradients – both in the temperature and in the density – and can cause therefore an overestimation of the mass. Locally, this can lead to a mass estimate up to two times the true mass. Substructure on the other hand tends to flatten the azimuthally averaged profile and hence leads to an underestimation of the mass, in extreme cases to deviations of 50% of the true mass (Schindler 1996).

In some cases, these deviations can be corrected for, e.g. in clusters in which substructures are well distinguishable, the disturbed part can be excluded from the mass analysis and a good mass estimate can be obtained. But in general, mass determinations in non-relaxed clusters should be done very cautiously.

Also the effect of magnetic fields on the mass determination was investigated. A considerable magnetic pressure compared to the thermal pressure would lead to an underestimation of the mass. Magneto-hydrodynamic simulations by Dolag et al. (1999) were used to perform the same comparison as mentioned above. Dolag & Schindler (2000) found that in relaxed clusters the mass is underestimated at most by a few percent and only in the cluster centre. In merging clusters, on the other hand, the mass can be strongly underestimated. The reason is that during the merger the gas is compressed and with it the magnetic field lines and hence the magnetic field is stronger and affects the mass determination.

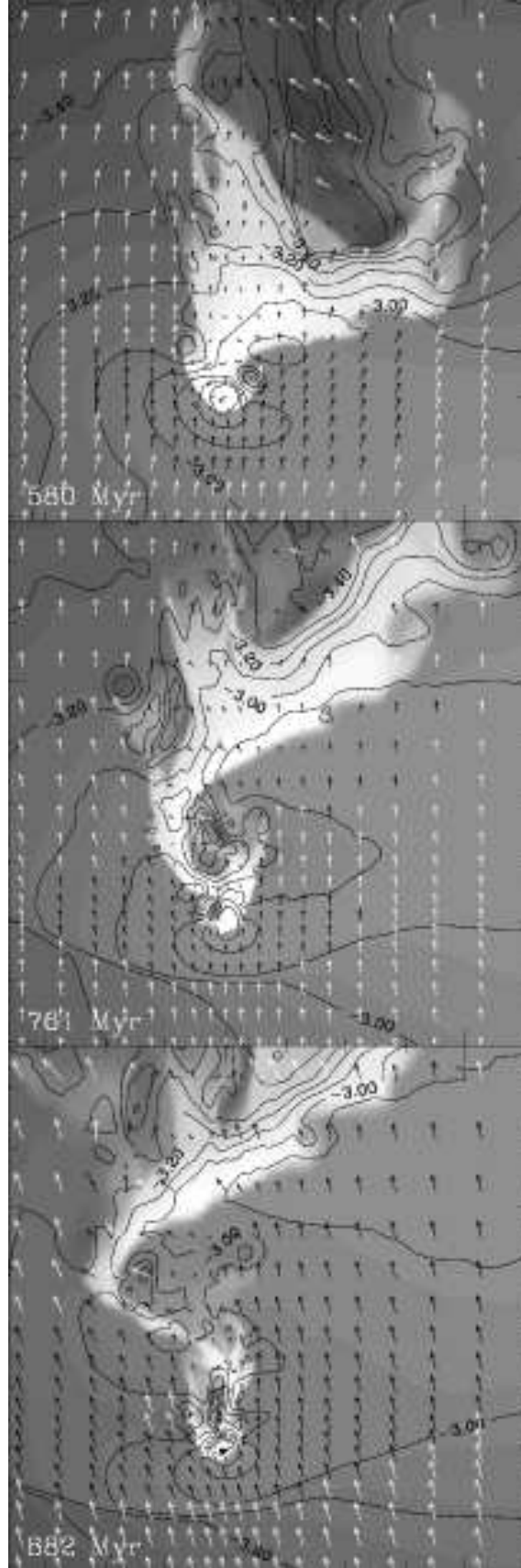


Figure 3: Gas density (grey scale) and pressure (contours) of a galaxy moving downwards towards the cluster centre. The arrows show the Mach vectors (white when $M > 1$, black otherwise). The gas of the galaxy is stripped due to ram pressure (from Toniazzo & Schindler (2001)).

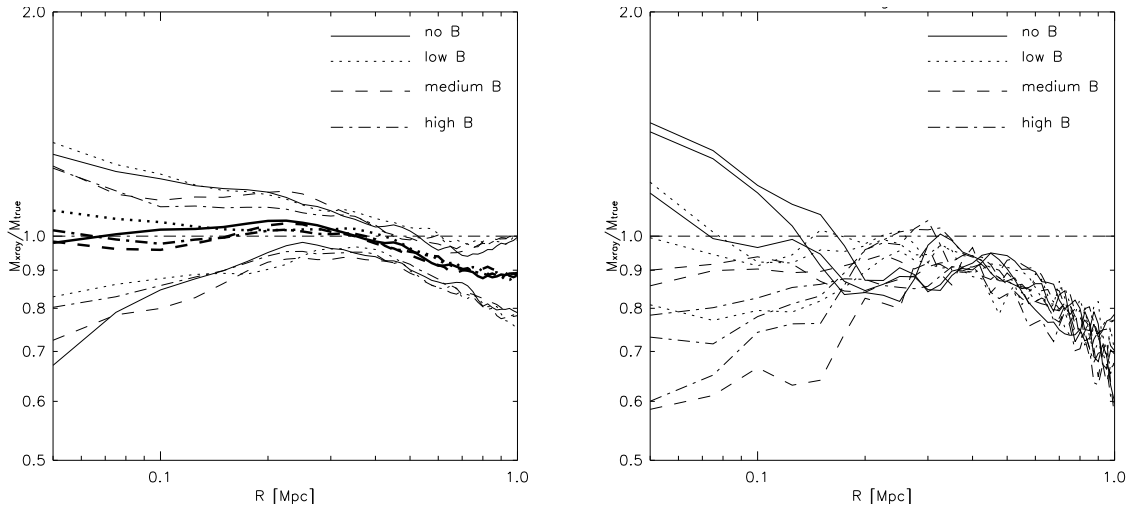


Figure 4: Left: Ratio of true and X-ray mass for 4 different magnetic field strengths. The bold lines are averaged profiles over 18 models, while the thin lines show the corresponding standard deviation. No dependence of the mass estimate on the magnetic field strength is visible. Right: Same for a cluster in the process of merging. For each model, the three projection directions are shown in the same line style. For this extreme merger model the magnetic effects are considerably larger than the usual scatter of mass profiles from different projection directions (from Dolag & Schindler (2000)).

8 Summary

Simulations provide a unique way to follow the dynamics of galaxy clusters. Different dynamical states can be distinguished. Each stage of a merger (pre-merger, collision, post-merger) is characterised by different observational characteristics. Therefore not only merger clusters can be distinguished from relaxed clusters but even the exact dynamical state can be determined by a detailed comparison of simulations and observations.

Numerical models can be used to test observational methods, like e.g. the mass determination or the temperature determination. They define the limitations of these methods and help improving them.

In the future not only the increase of resolution is important but also inclusion of physical effects, e.g. radiative cooling, star formation, and magnetic field. All these improvements together will yield more realistic models, that will help us to understand the physical processes in clusters and to make the best use of clusters as diagnostic tools for cosmology.

Acknowledgments

I would like to thank the organisers for organising this extremely interesting, fruitful and enjoyable conference.

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